

# Influence functional in two dimensional dilaton gravity

Fernando C. Lombardo <sup>\*</sup> and Francisco D. Mazzitelli <sup>†</sup>

*Departamento de Física, Facultad de Ciencias Exactas y Naturales*

*Universidad de Buenos Aires - Ciudad Universitaria, Pabellón I*

*1428 Buenos Aires, Argentina*

## Abstract

We evaluate the influence functional for two dimensional models of dilaton gravity. This functional is exactly computed when the conformal invariance is preserved, and it can be written as the difference between the Liouville actions on each closed-time-path branch plus a boundary term. From the influence action we derive the covariant form of the semiclassical field equations. We also study the quantum to classical transition in cosmological backgrounds. In the conformal case we show that the semiclassical approximation is not valid because there is no imaginary part in the influence action. Finally we show that the inclusion of the dilaton loop in the influence functional breaks conformal invariance and ensures the validity of the semiclassical approximation.

PACS numbers: 04.60Kz, 04.62+v, 05.40+j

---

<sup>\*</sup>Electronic address: lombardo@df.uba.ar

<sup>†</sup>Electronic address: fmazzi@df.uba.ar

## I. INTRODUCTION

In semiclassical and quantum gravity it is of interest to compute the backreaction of quantum fields on the spacetime geometry. One of the most interesting questions to answer in this context is about the endpoint of black hole evaporation and the information loss puzzle. It would also be useful to understand which is the effect of the quantum fluctuations on classical singularities of general relativity, in particular in cosmological settings.

The standard approach to this problem consists of studying the so called semiclassical Einstein equations, which include the quantum effects by taking as a source the quantum mean value of the energy momentum tensor of the quantum fields. The spacetime metric is considered a classical object. The analysis of the semiclassical equations in realistic models is a very difficult task. For this reason, it is of interest to analyze solvable toy models in which some of the difficulties are not present. Two dimensional dilaton gravity theories are very useful in this sense. From them one may take a better understanding of the main aspects about the quantum properties of black holes and the influence of quantum effects in cosmological situations [1].

The fundamental result of Callan et al [2] is that two dimensional gravity coupled to a dilaton field  $\phi$  and  $N$  conformal fields  $f_i$  contains black hole solutions and Hawking radiation. The backreaction of the quantum fields  $f_i$  can also be computed in this model. In particular, with minor modifications [3,4] the semiclassical problem can be completely solved. But *is the semiclassical approximation justified?* This is of course an old and important question for which, as we will show, two dimensional models also provide interesting simplifications.

After Hartle and Hawking proposal for the wave function of the Universe, the validity of the semiclassical approximation has been extensively studied, mainly in four dimensional, cosmological minisuperspace models. It was realized that the semiclassical limit is based on two main ingredients: correlations and decoherence [5]. The correlations between different variables was first analyzed using the Wigner function [6], while the decoherence was studied through the reduced density matrix [7]. Both ingredients are not independent: and excess

of decoherence can destroy the correlations [8]. The quantum to classical transition was subsequently analyzed using the decoherence functional of Hartle and Gell-Mann [9], which is a functional of two histories  $\mathcal{D}[g_{\mu\nu}^+, g_{\mu\nu}^-]$  after integration of the quantum variables. The metric  $g_{\mu\nu}$  can be described as a classical one if the decoherence functional is approximately diagonal. When this is the case, probabilities for different histories satisfy the sum rules, and then quantum effects are negligible.

A better understanding of this problem can be achieved by viewing it in the context of quantum open systems, where the metric  $g_{\mu\nu}$  is viewed as the “system” and the quantum fields as the “environment”. The influence functional [10] technique is the adequate tool to describe the effect of the environment, and provides information about dissipation, noise and the quantum to classical transition suffered by the system. The influence functional is closely related to the decoherence functional of Hartle and Gell-Mann, and also gives the temporal evolution of the reduced density matrix. In summary, is a fundamental tool to develop a systematic study of the validity of the semiclassical approximation.

In this paper, as a first step towards the analysis of the quantum to classical transition in two dimensional models, we will compute the influence functional by tracing out the quantum matter fields. We will show that these toy models are very useful to understand its main features since, due to conformal invariance, it can be computed exactly. We will use the general result for the influence functional to derive the semiclassical equations of the model and to discuss the validity of the semiclassical limit in the cosmological context. We will show that the semiclassical approximation is not justified in this case, unless conformal invariance is broken, and that this can be done by including the quantum fluctuations of the dilaton field (in this case the influence functional can be computed only perturbatively). We will also discuss the dependence of the influence functional with the matching hypersurface.

The paper is organized as follows. In the next section we introduce the model, the definition of the influence functional and its relation with the Close Time Path (CTP) effective action [11]. In Section III we derive the influence functional from the Euclidean effective action. In Sec. IV, we derive the semiclassical, covariant equations of motion from the CTP

effective action. Section V contains the complete evaluation of the influence functional for cosmological metrics. We will find that for  $t = \text{const}$  matching hypersurfaces, there is no loss of coherence. A discussion about an alternative choice of matching surfaces completes this section. In Sec. VI, we will show that the inclusion of the quantum fluctuations of the dilaton field breaks conformal invariance, and produces an explicit imaginary part in the influence functional that reveals the presence of particle creation and decoherence effects. Section VII contains our final remarks.

## II. THE MODEL

The classical 2D Callan-Giddings-Harvey-Strominger (CGHS) action is given by

$$S_{CGHS} = \frac{1}{2\pi} \int d^2x \sqrt{-g(x)} \{ e^{-2\phi} [R + 4(\nabla\phi)^2 + 4\lambda^2] - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \}, \quad (1)$$

where  $\phi$  is the dilaton field,  $R$  is the 2D Ricci scalar,  $\lambda$  is a positive constant, and the  $f_i$  are  $N$  massless scalar matter fields conformally coupled to the 2D geometry.

Considering the quantum effects of the scalar fields, the exact *Euclidean* effective action for the two dimensional model is [2]

$$S_{eff}^E = S_{CGHS}^E - \frac{N}{96\pi} \int d^2x \sqrt{g(x)} \int d^2x' \sqrt{g(x')} R(x) \frac{1}{\square} R(x'). \quad (2)$$

The second term in Eq. (2) is the Polyakov-Liouville effective action [12] derived from the trace anomaly of the massless scalar fields. This term is non local and contains the inverse of the d'lambertian, i.e. the two-point Euclidean propagator. In the conformal gauge this effective action can be written as

$$S_{eff}^E = S_{CGHS}^E + S_{Liouv}^E = S_{CGHS}^E - \frac{N}{12} \int d^2x \rho \partial_+ \partial_- \rho. \quad (3)$$

It is usually claimed that the semiclassical field equations can be derived from the effective action (2). Strictly speaking, this is not correct. Replacing the Euclidean propagator by the Feynman one, one obtains the usual *in-out* effective action. As is very well known, the effective equations derived from this action are neither real nor causal because they are

equations for *in-out* matrix elements and not for mean values. The solution to this problem is also well known. Using the CTP formalism [13] one can construct an *in-in* effective action that produces real and causal field equations for *in-in* expectation values. The effective action can be written as (we are assuming a semiclassical point of view because we are not integrating over the metric configurations)

$$e^{iS_{eff}^{CTP}[g^\pm, f^\pm]} = \mathcal{N} e^{i(S_{CGHS}[g^+, f^+] - S_{CGHS}[g^-, f^-])} \int \mathcal{D}\hat{f}^+ \mathcal{D}\hat{f}^- e^{i(S_{matter}[g^+, \hat{f}^+] - S_{matter}[g^-, \hat{f}^-])}. \quad (4)$$

The field equations are obtained taking the variation of this action with respect to the  $g_{\mu\nu}^+$  metric, and then setting  $g_{\mu\nu}^+ = g_{\mu\nu}^-$ . The integration in Eq. (4) is over quantum fluctuations around the background matter fields.  $\hat{f}^+$  and  $\hat{f}^-$  must contain negative and positive frequency modes, respectively, in the remote past (these are the *in* boundary conditions) and must coincide at a finite future spacelike hypersurface. This hypersurface, that must be a Cauchy hypersurface, will be denoted by  $\Sigma$ . The path integral can be thought as the path sum of two different fields evolving in two temporal branches; one going forward in time in the presence of source  $g_{\mu\nu}^+$  from the *in* vacuum to  $\Sigma$  and the other backward in time in the presence of  $g_{\mu\nu}^-$  from  $\Sigma$  to the *in* vacuum. The constraint that must be imposed is  $\hat{f}^+|_\Sigma = \hat{f}^-|_\Sigma$ . We stress that  $\hat{f}$  and  $g_{\mu\nu}$  are independent on the  $+$  and  $-$  branches.

It can be easily proved that the CTP-effective action takes the form

$$S_{eff}^{CTP} = S_{CGHS}[g_{\mu\nu}^+, f^+] - S_{CGHS}[g_{\mu\nu}^-, f^-] + \Gamma_{IF}[g_{\mu\nu}^\pm]. \quad (5)$$

The functional  $\Gamma_{IF}[g_{\mu\nu}^\pm]$  is so called *influence action* [13]. Note that it depends (non trivially, as we will see) on the matching hypersurface  $\Sigma$ . From the quantum open systems point of view, after integration of the “environment” (the quantum fluctuations of the matter fields  $\hat{f}_i$ ), one ends up with an effective theory for the “system” (the metric  $g_{\mu\nu}$ , the dilaton, and the classical background of the matter fields,  $f_i$ ). The quantity  $e^{iS_{eff}^{CTP}}$  is the influence functional and coincides with the *decoherence functional* of Hartle and Gell-Mann [9].

In our present case, we are choosing an initial condition such that the initial quantum state for the scalar fields is the *in* vacuum state. With this particular choice there is a simple

relation between the influence functional and the effective action, as can be seen from Eq. (5). In general, the influence functional is a more complicated object that strongly depends of the initial conditions [14].

It is interesting to note that the influence functional can be written as

$$e^{iS_{eff}^{CTP}} = \sum_{\alpha} \langle 0, in | \alpha, T \rangle_{g^-} \langle \alpha, T | 0, in \rangle_{g^+}, \quad (6)$$

therefore, it can be interpreted as the scalar product on  $\Sigma$  between the states constructed as temporal evolutions (on the two different metrics  $g_{\mu\nu}^{\pm}$ ) from the common *in* state up to the future hypersurface  $\Sigma$ . The CTP-effective action can alternatively written in terms of the Bogolubov coefficients connecting the *in* and *out* basis in each temporal branch. This implies that there is decoherence if and only if there is particle creation during the field evolution [15].

### III. TWO DIMENSIONAL INFLUENCE FUNCTIONAL

In an alternative, and more concise notation, we can write the effective action of Eq. (4) as [16,17]

$$e^{iS_{eff}^C[g,f]} = \mathcal{N} e^{iS_{GHS}^C[g,f]} \int \mathcal{D}\hat{f} e^{iS_{matter}^C[g,\hat{f}]}, \quad (7)$$

where we have introduced the CTP complex temporal path  $\mathcal{C} = \mathcal{C}_+ \cup \mathcal{C}_-$ , going from minus infinity to  $\Sigma$  ( $\mathcal{C}_+$ ), and backwards, with a decreasing (infinitesimal) imaginary part ( $\mathcal{C}_-$ ). Time integration over the contour  $\mathcal{C}$  is defined by  $\int_{\mathcal{C}} dt = \int_{\mathcal{C}_+} dt - \int_{\mathcal{C}_-} dt$ . The field fluctuation  $\hat{f}$  appearing in Eq. (7) is related to those in Eq. (4) by  $\hat{f}(t, x) = \hat{f}_{\pm}(t, x)$  if  $t \in \mathcal{C}_{\pm}$ . The same applies to  $g_{\mu\nu}$  and to the background  $f$ . This equation is useful because it has the structure of the usual *in-out* or the Euclidean effective action. Feynman rules are therefore the ordinary ones, replacing Euclidean propagator by

$$G(x, y) = \begin{cases} G_{++}(x, y) = i\langle 0, in | T \hat{f}^+(x) \hat{f}^+(y) | 0, in \rangle, & t, t' \text{ both on } \mathcal{C}_+ \\ G_{--}(x, y) = -i\langle 0, in | \tilde{T} \hat{f}^-(x) \hat{f}^-(y) | 0, in \rangle, & t, t' \text{ both on } \mathcal{C}_- \\ G_{+-}(x, y) = -i\langle 0, in | \hat{f}^+(x) \hat{f}^-(y) | 0, in \rangle, & t \text{ on } \mathcal{C}_+, t' \text{ on } \mathcal{C}_- \\ G_{-+}(x, y) = i\langle 0, in | \hat{f}^-(y) \hat{f}^+(x) | 0, in \rangle, & t \text{ on } \mathcal{C}_-, t' \text{ on } \mathcal{C}_+ \end{cases} \quad (8)$$

It is important to note that each propagator has been defined taking into account that the field fluctuations correspond to different metrics. For example,  $G_{++}$  is the time ordered product for both fields in the  $+$  metric, while  $G_{--}$  is the anti-temporal ordered product for fields in the  $-$  metric. Therefore, in this case it is not true that  $G_{++} = G_{--}^*$  because each propagator is defined on a different metric. In a similar way,  $G_{+-}$  contains one field on the  $+$  metric and the other one on the  $-$  metric. Moreover, the Feynman and Dyson propagators can be expressed as

$$G_{\pm\pm}(x, y) = G_{\pm}(x, y) \theta(x^0 - y^0) + G_{\mp}(x, y) \theta(y^0 - x^0) \quad (9)$$

but now it is not true that  $G_+ = G_{+-}$  and  $G_- = G_{-+}$ .

From Eqs (2) and (8) the closed-time-path effective action can be written as

$$S_{eff}^{CTP} = S_{CGHS}(g_{\mu\nu}^+, f^+) - S_{CGHS}(g_{\mu\nu}^-, f^-) - \frac{N}{96\pi} \int d^2x \sqrt{-g(x)} \int d^2y \sqrt{-g(y)} R^a(x) G_{ab}(x, y) R^b(y), \quad (10)$$

where the indices  $a$  and  $b$  are denoting each CTP branch  $+$  and  $-$ . From this expression is possible, at least formally, to write the real and imaginary parts of the influence action for two generic  $+/-$  metrics.

It is instructive to analyze the influence functional in the conformal gauge

$$g_{+-} = -\frac{1}{2}e^{2\rho} \quad , \quad g_{--} = g_{++} = 0. \quad (11)$$

The Ricci scalar is  $R = -8e^{-2\rho} \partial_+ \partial_- \rho$ , where  $\partial_{\pm}$  denotes derivatives with respect to the coordinates  $x^{\pm} = (x^0 \pm x^1)$ . In this gauge, the closed-time-path-effective action reads

$$S_{eff}^{CTP}[\rho^+, f^+, \rho^-, f^-; \Sigma] = S_{CGHS}(\rho^+, f^+) - S_{CGHS}(\rho^-, f^-) - \frac{N}{6\pi} \int d^2x \int d^2y \partial_+ \partial_- \rho^a(x^{\pm}) G_{ab}(x^{\pm}, y^{\pm}) \partial_+ \partial_- \rho^b(y^{\pm}). \quad (12)$$

After integrations by parts, the effective action can be expressed as the classical terms, plus the difference between the Liouville action in each metric, plus boundary terms

$$\begin{aligned}
S_{eff}^{CTP}[\rho^+, f^+, \rho^-, f^-; \Sigma] &= S_{CGHS}(\rho^+, f^+) - S_{CGHS}(\rho^-, f^-) \\
&\quad - \frac{N}{12\pi} \int d^2x [\rho^+ \partial_+ \partial_- \rho^+ - \rho^- \partial_+ \partial_- \rho^-] \\
&\quad - \frac{N}{6\pi} \{\text{boundary terms}\},
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
\{\text{boundary terms}\} &= \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \left[ \partial_{x-} \Delta(x, k(x)) N_1[x, k(x); y, \bar{k}(y)] \partial_{y-} \Delta(y, \bar{k}(y)) \right. \\
&\quad + 2 \partial_{x-} \Xi(x, k(x)) N_2[x, k(x); y, \bar{k}(y)] \partial_{y-} \Delta(y, \bar{k}(y)) \\
&\quad + \partial_{x-} \Xi(x, k(x)) N_3[x, k(x); y, \bar{k}(y)] \partial_{y-} \Xi(y, \bar{k}(y)) \left. \right] \\
&\quad - 2 \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \left[ \Xi(x, k(x)) \partial_{x+} N_2[x, k(x); y, \bar{k}(y)] \partial_{y-} \Delta(y, \bar{k}(y)) \right. \\
&\quad + \Xi(x, k(x)) \partial_{x+} N_4[x, k(x); y, \bar{k}(y)] \partial_{x-} \Xi(y, \bar{k}(y)) \\
&\quad \left. - \frac{1}{2} \Xi(x, k(x)) \partial_{x+} \partial_{y+} N_4[x, k(x); y, \bar{k}(y)] \Xi(y, \bar{k}(y)) \right].
\end{aligned} \tag{14}$$

The matching hypersurface  $\Sigma$  is defined by  $t_x = k(x)$ ,  $t_y = \bar{k}(y)$ ;  $\Delta = \frac{1}{2}(\rho^+ - \rho^-)$ ,  $\Xi = \frac{1}{2}(\rho^+ + \rho^-)$ , and

$$\begin{aligned}
N_1 &= G_{++} + G_{+-} - G_{-+} - G_{--} \\
N_2 &= G_{++} + G_{+-} + G_{-+} + G_{--} \\
N_3 &= G_{++} - G_{+-} - G_{-+} + G_{--} \\
N_4 &= G_{++} - G_{+-} + G_{-+} - G_{--}.
\end{aligned} \tag{15}$$

The expression (13) for the effective action is absolutely general, and can be applied to any particular metric in the conformal gauge. It is worth to note that if the influence action contains a non-trivial imaginary contribution, it must be included in the boundary terms. If both metrics  $g_{\mu\nu}^\pm$  coincide asymptotically in the future, and if the matching hypersurface is within such region, all the boundary terms vanish because the usual relations between



Green functions are valid as in the remote past;  $N_4$  and  $\Delta$  are simultaneously zero and only the trace anomaly survives.

The main result of this Section is that the influence action can be easily computed from the Euclidean effective action. The matching hypersurface  $\Sigma$  plays a crucial role in its evaluation. In particular, in the conformal gauge all the relevant information about the quantum to classical transition is contained in a  $\Sigma$ -dependent boundary term.

#### IV. THE COVARIANT EQUATIONS OF MOTION

Although we are mainly interested in the analysis of the quantum to classical transition in the two dimensional models, in this Section we will derive the covariant field equations from the CTP effective action. In previous works [18], it was claimed that the semiclassical equations of motion follow from the usual effective action Eq. (2). As we already pointed out, strictly speaking this is not correct, since there is no variational principle for the initial-value problem, unless one uses the CTP formalism. Indeed, the semiclassical field equations have been previously obtained from the classical equations by taking as a source the quantum mean value of the energy-momentum tensor, i.e.,

$$2\pi \frac{\delta S_{CGHS}}{\delta g_{\mu\nu}} = \langle T_{\mu\nu} \rangle. \quad (16)$$

The components of the energy-momentum tensor have been derived from the trace anomaly and the imposition of the conservation laws  $\langle T^{\mu\nu} \rangle_{;\nu} = 0$  [19]. In all previous works the field equations have been written and analyzed in the conformal gauge. Moreover, with this procedure it is possible to get the field equations only for conformal matter fields, since for massive or non conformally coupled fields the trace of the energy momentum tensor is not known a priori.

The CTP formalism allow us to derive the covariant equations of motion from

$$\frac{\delta S_{eff}^{CTP}}{\delta g_{\mu\nu}^+} \Big|_{g_{\mu\nu}^+ = g_{\mu\nu}^-} = 0. \quad (17)$$

At this point, the only difficulty resides in the knowledge of the functional variation of the Green functions with respect to the metric. From the definition of the Green functions, and after expanding the field in modes we can prove that [20]

$$\delta G_{++} = G_{ret} \delta \square G_{++} + G_{++} \delta \square G_{adv} - G_{ret} \delta \square G_{adv}, \quad (18)$$

$$\delta G_{+-} = G_{ret} \delta \square G_{+-}, \quad (19)$$

$$\delta G_{-+} = G_{ret} \delta \square G_{-+}, \quad (20)$$

where  $G_{ret}$  and  $G_{adv}$  are the usual retarded and advanced Green functions, and where

$$\delta \square = -\nabla^\mu \nabla^\nu \delta g_{\mu\nu} - \frac{1}{2} \partial^\lambda g^{\mu\nu} (\delta g_{\lambda\nu;\mu} + \delta g_{\mu\lambda;\nu} - \delta g_{\mu\nu;\lambda}). \quad (21)$$

In Eqs. (18) - (21) all the propagators  $G_{ret}$ ,  $G_{adv}$ ,  $G_{+-}$ , and  $G_{-+}$  are evaluated at  $g_{\mu\nu}^+ = g_{\mu\nu}^-$ , since this is all we need to obtain the field equations (see Eq. (17)).

After some algebra, the covariant equations of motion can be written as

$$\begin{aligned} \frac{\delta S_{CGHS}}{\delta g_{\mu\nu}^+} \equiv & \frac{1}{2\pi} \langle T_{\mu\nu} \rangle = \\ & - \frac{N}{48\pi} \int d^2 y \sqrt{-g(y)} R(y) [\nabla_\mu \nabla_\nu - g_{\mu\nu} \square]_{(x)} G_{ret}(x, y) \\ & + \frac{N}{192\pi} \int d^2 x \sqrt{-g(x)} \int d^2 y \sqrt{-g(y)} \{ 2R(x) G_{ret}(x, z) \\ & \times [\nabla_\mu \nabla_\nu - \frac{1}{2} g_{\mu\nu}(z) \square]_{(z)} G_{ret}(z, y) R(y) \\ & - 2R(x) \partial_\mu(z) G_{ret}(x, z) \partial_\nu(z) G_{ret}(z, y) R(y) \\ & + R(x) g_{\mu\nu}(z) \partial^\alpha(z) G_{ret}(x, z) \partial_\alpha(z) G_{ret}(z, y) R(y) \}. \end{aligned} \quad (22)$$

Note that these are non-local, real, and causal equations of motion. From these equations we can calculate the trace of the stress tensor

$$\langle T_\mu^\mu \rangle = 2\pi g^{\mu\nu} \frac{\delta S_{eff}^{CTP}}{\delta g_{\mu\nu}^+} = N \frac{R}{24}, \quad (23)$$

which gives the well known trace anomaly [21].

To make contact with previous works, we write the covariant equations of motion in the conformal gauge. Writting the curvature scalar in this gauge, the components of the stress tensor read

$$\langle T_{+-} \rangle = \langle T_{-+} \rangle = -\frac{N}{12} \partial_+ \partial_- \rho, \quad (24)$$

$$\langle T_{\pm\pm} \rangle = -\frac{N}{12} [\partial_{\pm}^2 \rho - \partial_{\pm} \rho \partial_{\pm} \rho - t^{\pm}]. \quad (25)$$

The functions  $t^{\pm}$  depend on  $x^{\pm}$  and can be expressed as

$$t^{\pm} = \partial_{\pm}^2 S^{\pm} - 2\partial_{\pm} S^{\pm} \partial_{\pm} S^{\pm}, \quad (26)$$

where functions  $S^{\pm}$  are given by

$$S^+(x^+) = \int d^2 y \partial_{y^-} [\rho(y) \partial_{y^+} G_{ret}(x, y)], \quad (27)$$

$$S^-(x^-) = \int d^2 y \partial_{y^+} [\partial_{y^-} \rho(y) G_{ret}(x, y)]. \quad (28)$$

Of course, Eqs. (24) and (25) coincide with the results obtained in previous works [2,3] using a different method.

The functions  $t^{\pm}$  depend on the quantum state of the matter fields. In our case, they are completely determined by the boundary conditions in the remote past that we used to compute the CTP effective action (see Eq. (4)), and correspond to the *in* vacuum state. As an example, we will obtain the explicit expression of these functions for cosmological metrics, and for a collapsing matter wave.

For cosmological metrics,  $\rho$  is a function of the conformal time  $t$ ,  $\rho = \rho(t)$ . Then using that the retarded propagator is given by

$$G_{ret}(x, y) = \theta(t_x - t_y - |x - y|), \quad (29)$$

we can compute the functions  $S^{\pm}$  and  $t^{\pm}$ , that in this case are

$$t^{\pm} = \partial_{\pm}^2 \rho - \partial_{\pm} \rho \partial_{\pm} \rho. \quad (30)$$

Therefore, from Eqs. (24) and (25) we obtain

$$\langle T_{+-} \rangle = \langle T_{-+} \rangle = -\frac{N}{12}\ddot{\rho}, \quad (31)$$

$$\langle T_{\pm\pm} \rangle = 0. \quad (32)$$

Only the trace anomaly survives [22].

Let us now consider the well known case of an  $f$  shock wave traveling in the  $x^- = t - x$  direction described by the stress tensor

$$\frac{1}{2}\partial_+ f \partial_+ f = a\delta(x^+ - x_0^+), \quad (33)$$

where  $a$  is a positive constant. The classical metric is given by

$$e^{-2\rho} = e^{-2\phi} = -a(x^+ - x_0^+)\theta(x^+ - x_0^+) - \lambda^2 x^+ x^-. \quad (34)$$

For  $x^+ < x_0^+$ , this is simply the linear dilaton vacuum, while for  $x^+ > x_0^+$  there is a black hole with mass  $ax_0^+\lambda$ .

If we introduce the “tortoise” coordinates  $\lambda\sigma^\pm$

$$\lambda x^+ = e^{\lambda\sigma^+} \quad , \quad \lambda x^- = -(e^{-\lambda\sigma^-} + \frac{a}{\lambda}), \quad (35)$$

the metric can be written as

$$e^{-2\rho} = \begin{cases} [1 + (\frac{a}{\lambda})e^{\lambda\sigma^-}]^{-1}, & \text{if } \sigma^+ < 0 \\ \{1 + (\frac{a}{\lambda})e^{[\lambda(\sigma^- - \sigma^+)]}\}^{-1}, & \text{if } \sigma^+ > 0 \end{cases}, \quad (36)$$

where we have set  $\lambda x_0 = 1$  for simplicity.

The retarded propagator formally has the same structure than before but now in the tortoise coordinates. Therefore the functions  $t^\pm$  read

$$t^\pm = \partial_{\sigma^\pm}^2 \rho - \partial_{\sigma^\pm} \rho \partial_{\sigma^\pm} \rho, \quad (37)$$

and the components of the energy-momentum tensor are

$$\langle T_{\pm\pm} \rangle = -\frac{N}{12}[\partial_{\pm}^2 \rho - \partial_{\pm} \rho \partial_{\pm} \rho - \partial_{\sigma^\pm}^2 \rho + \partial_{\sigma^\pm} \rho \partial_{\sigma^\pm} \rho]. \quad (38)$$

This result is valid for all the spacetime. In particular, in the vacuum region  $\partial_{\sigma^+}\rho$  vanishes and one obtains

$$t^+(\sigma^+) = 0. \quad (39)$$

From Eq. (36) we find

$$t^-(\sigma^-) = -\frac{1}{4}\lambda^2 \left[ 1 - \left( 1 + \frac{a}{\lambda} e^{\lambda\sigma^-} \right)^{-2} \right]. \quad (40)$$

which coincides with previous results [2].

In this section we have presented a derivation of the semiclassical field equations from the CTP effective action. The covariant version of these equations contains non-local terms, that become local in the conformal gauge.

## V. INFLUENCE FUNCTIONAL FOR COSMOLOGICAL HISTORIES

The goal of this section will be to write the influence action for two cosmological spacetimes and evaluate explicitly its imaginary part. The influence functional is a matching hypersurface dependent object. We will see that the choice of this hypersurface will be determinant for decoherence phenomena.

At this point, it is necessary to state precisely what does a “matching hypersurface” mean. We will follow closely Ref. [23]. Let us denote by  $\mathcal{M}$  and  $\tilde{\mathcal{M}}$  the spacetimes described by metrics  $g_{\mu\nu}^+$  and  $g_{\mu\nu}^-$  respectively. We are assuming that both spacetimes are asymptotically flat in the past, and that they “share” a spacelike hypersurface  $\Sigma$ . We can always define a hypersurface  $\Sigma_{\mathcal{M}}$  in the spacetime  $\mathcal{M}$  through a relation between  $t$  and  $x$ , say  $t = k(x)$ . We can also define a hypersurface  $\Sigma_{\tilde{\mathcal{M}}}$  in  $\tilde{\mathcal{M}}$  by  $\bar{t} = \bar{k}(\bar{x})$ . In order to identify  $\Sigma_{\mathcal{M}}$  and  $\Sigma_{\tilde{\mathcal{M}}}$  in a common hypersurface  $\Sigma$ , we must introduce a map between points on both hypersurfaces which follows from identifying the local intrinsic geometry. In two dimensional dilaton gravity models, an invariant definition of a one-geometry is provided by the value of the dilaton field  $\phi(s)$ , as a function of the proper distance along the hypersurface. The identification of one-geometries therefore implies that for the same proper

distance (measured with respect to an arbitrary reference point)  $ds^2 = d\bar{s}^2$ , the dilaton field must have the same value for each geometry on  $\Sigma$ , i.e.,  $\phi^+(s) = \phi^-(\bar{s})$ . Then it follows that  $d\phi^+/ds = d\phi^-(\bar{s})/d\bar{s}$ .

Given both spacetimes and the function  $k$  that defines the hypersurface  $\Sigma_{\mathcal{M}}$  in  $\mathcal{M}$ , the conditions imposed by the identification allow us to determine the function  $\bar{k}$ , and therefore the hypersurface  $\Sigma_{\tilde{\mathcal{M}}}$  in  $\tilde{\mathcal{M}}$ . If the equations have real solution for the function  $\bar{k}$ , then the hypersurface  $\Sigma_{\mathcal{M}}$  “fits” in  $\tilde{\mathcal{M}}$ .

Let us now consider two cosmological metrics characterized by the functions  $\rho^+(t)$  and  $\rho^-(t)$ . The starting point to compute the influence functional is the evaluation of the Green functions  $G_{ab}$ . Both metrics are asymptotically flat in the past and conformal to Minkowski spacetime (everywhere). Therefore, the propagators in the *in* vacuum state have the same functional structure than in flat spacetime. For example, the Feynman propagator is given by

$$\begin{aligned} G_{++}(x, y) &= i\langle 0, in | T \hat{f}^+(x) \hat{f}^+(y) | 0, in \rangle \\ &= \frac{1}{2\pi^2} \int d^2p \frac{e^{ip(x-y)}}{p^2 + i\epsilon} = -\frac{2\pi i}{2\pi^2} \int_0^\infty \frac{dp}{p} e^{-ip(x-y)} e^{-ip|t_x - t_y|} \\ &= \frac{\pi}{2} \text{Sgn}[|t_x - t_y| + x - y] - i \text{Log}[|t_x - t_y| + x - y] + C, \end{aligned} \quad (41)$$

where  $C$  is an indeterminate constant (this indetermination comes from the infrared divergence at  $p \rightarrow 0$ ). Similar expressions hold for the other propagators. It is important to note that in  $G_{+-}(x, y)$  and  $G_{-+}(x, y)$  the coordinates  $x$  and  $y$  correspond to different spacetimes.

### A. Constant time hypersurface

Let us consider a constant time hypersurface  $\Sigma_{\mathcal{M}}$  in  $\mathcal{M}$ , defined as  $t = T$ . To make the embedding we must impose  $\phi^+ = \phi^-$  on  $\Sigma$ . As  $\phi^-$  is constant on  $\Sigma_{\tilde{\mathcal{M}}}$ , this hypersurface must also be of constant time  $\bar{t} = \bar{T}$ . After a shift of the time coordinate we can set  $\bar{T} = T$ .

The CTP-effective action for cosmological metrics can be written as

$$S_{eff}^{CTP} = S_{CGHS}(\rho^+, f^+) - S_{CGHS}(\rho^-, f^-)$$

$$- \frac{N}{6\pi} \int_{-\infty}^T dt_x \int_{-\infty}^T dt_y \ddot{\rho}^a(t_x) \ddot{\rho}^b(t_y) \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy G_{ab}(x, y), \quad (42)$$

where  $a$  and  $b$  denote the CTP branches again. We must compute the spatial integral of the propagator. Using dimensional regularization, we find

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy G_{++} = \frac{\Omega}{2} |t_x - t_y|, \quad (43)$$

where  $\Omega$  is a global volume factor. Similar expressions hold for  $G_{--}$  and  $G_{+-}$ .

Replacing Eq. (43) into Eq. (42), is possible to show, after some integrations by parts, that the CTP-effective action for cosmological metrics is given by

$$\begin{aligned} S_{eff}^{CTP} &= S_{CGHS}(\rho^+, f^+) - S_{CGHS}(\rho^-, f^-) \\ &\quad - \frac{N}{12\pi} \int d^2x [\rho^+ \partial_+ \partial_- \rho^+ - \rho^- \partial_+ \partial_- \rho^-]. \end{aligned} \quad (44)$$

As is immediately noted, there is not any imaginary and/or non-local term in this action. The only correction to the classical term comes from the trace anomaly. The consequence of this fact is that the decoherence functional is identically one. For the semiclassical approximation to be valid, the decoherence functional must be diagonal for macroscopically different spacetime geometries, even if they coincide on a single spacelike hypersurface. Therefore, we conclude that, due to conformal invariance, the two dimensional cosmological models do not have a well defined semiclassical limit. In order to obtain such a limit, it is necessary to break conformal invariance, as we will see in Section VI.

## B. More general hypersurfaces

In order to show explicitly the dependence of the results with the matching hypersurface, we will evaluate the influence functional for more general hypersurfaces. We will show that, even though the action is conformally invariant, imaginary terms do appear for some hypersurfaces.

We must compute

$$\int dx \int dy \int_{-\infty}^{\Sigma} dt_x \int_{-\infty}^{\Sigma} dt_y \ddot{\rho}^a(t_x) \ddot{\rho}^b(t_y) G_{ab}(x, y), \quad (45)$$

where we have defined the hypersurfaces in each branch by

$$k(x) = T + \Delta k^+(x), \quad (46)$$

and

$$\bar{k}(x) = T + \Delta k^-(x). \quad (47)$$

We will consider that  $\Delta k^+(x)$  and  $\Delta k^-(x)$  are small fluctuations around the hypersurface  $t = T$  and we will compute the influence functional up to second order in an expansion in powers of the fluctuations. It is obvious that the zeroth order gives only the trace anomaly. To first order the influence functional develops real terms<sup>1</sup>. The second order contribution is given by

$$\ddot{\rho}^a(T) \ddot{\rho}^b(T) \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy G_{ab}(x, T; y, T) \Delta k^a(x) \Delta k^b(y). \quad (48)$$

Introducing the corresponding propagators and performing the spatial integrations, we find an imaginary part proportional to

$$4\pi i \int_0^{+\infty} \frac{dp}{p} \left[ \ddot{\rho}^{2+}(T) \Delta k^{+2}(p) - \ddot{\rho}^+(T) \ddot{\rho}^-(T) \Delta k^{+*}(p) \Delta k^-(p) - \ddot{\rho}^-(T) \ddot{\rho}^+(T) \Delta k^+(p) \Delta k^{-*}(p) + \ddot{\rho}^{-2}(T) \Delta k^{-2}(p) \right], \quad (49)$$

where  $\Delta k^+(p)$  and  $\Delta k^-(p)$  denote the Fourier transforms of the perturbation functions.

The basic equations describing the embedding of  $\Sigma$  are

$$\phi^+[k(x)] = \phi^+[T + \Delta k^+(x)] = \phi^-[\bar{k}(y)] = \phi^-[T + \Delta k^-(y)]. \quad (50)$$

This identification may be described by the function  $y(x)$  between coordinates on  $\Sigma$  in each of the spacetimes. To complete the embedding we must impose the intervals in each spacetime to be the same on  $\Sigma$ . Therefore

---

<sup>1</sup>Note that these are boundary terms and therefore do not contribute to the equations of motion (see Eq. (13))



$$\left[\frac{dx}{dy}\right]^2 = \frac{1 - \left(\frac{d\bar{k}}{dy}\right)^2 e^{\rho^-[\bar{k}(y)]}}{1 - \left(\frac{dk}{dx}\right)^2 e^{\rho^+[k(x)]}}. \quad (51)$$

Expanding Eq. (50) for small  $\Delta k^+(p)$  and  $\Delta k^-(p)$ , and taking into account that  $y = x + \mathcal{O}(\Delta k^2)$ , we find that

$$\Delta k^+(x) \cong \Delta k^-(y) \frac{\dot{\phi}^-(T)}{\dot{\phi}^+(T)} \cong \Delta k^-(x) \frac{\dot{\phi}^-(T)}{\dot{\phi}^+(T)}. \quad (52)$$

Replacing Eq. (52) into (49) the imaginary term can be written as

$$4\pi i \left[ \ddot{\rho}^+(T) - \frac{\dot{\phi}^+(T)}{\dot{\phi}^-(T)} \ddot{\rho}^-(T) \right]^2 \int_0^\infty \frac{dp}{p} |\Delta k^+(p)|^2. \quad (53)$$

Therefore, there will be decoherence coming from the small fluctuations of the hypersurface around the  $t = T$  one, since the absolute value of the decoherence functional is given by

$$|\mathcal{D}[\rho^+, \rho^-; \Sigma]| \approx e^{-4\pi \left[ \ddot{\rho}^+(T) - \frac{\dot{\phi}^+(T)}{\dot{\phi}^-(T)} \ddot{\rho}^-(T) \right]^2 \int_0^\infty \frac{dp}{p} |\Delta k^+(p)|^2}. \quad (54)$$

The physical interpretation of the results found in this section is the following. The *in* quantum state of matter is the conformal vacuum. For  $t = T$  hypersurfaces, one can choose the *out* basis to be the conformal vacuum on both spacetimes. Therefore, the Bogolubov coefficients between *in* and *out* basis are trivial in both geometries. The influence functional is real and there is no decoherence. For more general hypersurfaces, one can choose as *out* basis the conformal vacuum in one of the spacetimes, but this basis in general do not correspond to the conformal vacuum in the other. Therefore, the *in* and *out* basis are essentially different in this spacetime, there is particle creation, and therefore decoherence.

We have shown that the influence functional has an imaginary part for some hypersurfaces, and that this imaginary contribution vanishes for the most common hypersurfaces of constant time. As a consequence, the absolute value of the decoherence functional also depends on the hypersurface.

## VI. THE DILATON LOOP

In the previous sections we were using the exact effective action for two dimensional dilaton gravity where only the scalar matter fields were quantized. In order to have a more

complete information about the quantum effects, we must consider the quantum fluctuations of the dilaton and the metric. There are some previous works where these effects have been taken into account, in the *in-out* effective action [24]. In this section we will compute the quantum correction coming from the dilaton field. This correction will be evaluated up to one loop and to lowest order in a covariant expansion in powers of the curvature.

Starting with the classical CGHS action, we split the dilaton field in a classical background and a small quantum fluctuation

$$\phi(x) = \phi_0(x) + \hat{\phi}(x), \quad (55)$$

introducing this splitting in the classical action, and dropping the linear terms in the fluctuation, we obtain up to quadratic order

$$\begin{aligned} S_\phi = \frac{1}{2\pi} \int d^2x \sqrt{-g(x)} \{ e^{-2\phi_0} [R + 4(\nabla\phi_0)^2 + 4\lambda^2] \\ + 4(\nabla\psi)^2 + 2[R(x) + 2(\nabla\phi_0)^2 + 4\nabla^2\phi_0] \psi^2 + 8\lambda^2\psi^2 \}, \end{aligned} \quad (56)$$

where we have redefined the dilaton field as  $\psi = e^{-\phi_0}\hat{\phi}$ . The action for the dilaton fluctuations  $\psi$  corresponds to that of a massive scalar field, non conformally coupled to the curvature and coupled to the dilaton background. Note that this action has a global minus sign with respect to the usual action for scalar fields.

The Euclidean effective action can be evaluated up to quadratic order in a covariant expansion in powers of the curvature [25] as

$$S_{eff}^\phi = \frac{1}{8\pi} \int d^2x \sqrt{g(x)} [PF_1[\square]P - 2PF_2[\square]R + R_{\mu\nu}F_3[\square]R^{\mu\nu} + RF_4[\square]R], \quad (57)$$

where the form factors are given by

$$F_1 = \frac{1}{2} \int_0^1 d\gamma G_\gamma^E, \quad (58)$$

$$F_2 = \frac{1}{2} \int_0^1 d\gamma \frac{(1-\gamma^2)}{4} G_\gamma^E, \quad (59)$$

$$F_3 = \frac{1}{2} \int_0^1 d\gamma \frac{\gamma^4}{6} G_\gamma^E, \quad (60)$$

$$F_4 = \frac{1}{2} \int_0^1 d\gamma \frac{(3 - 6\gamma^2 - \gamma^4)}{48} G_\gamma^E, \quad (61)$$

$G_\gamma^E = \left[ \frac{32\lambda^2}{1-\gamma^2} - \square \right]^{-1}$  is the Euclidean massive propagator, and  $P$  takes into account the nonconformal coupling  $P = 2R + 4(\nabla\phi_0)^2 + 8\nabla^2\phi_0$ . Note that, as conformal invariance is broken, it is no longer possible to compute the effective action exactly. This effective action has a limited region of validity. It is applicable only under the condition  $\nabla\nabla \mathcal{R} \gg \mathcal{R}^2$ , where the symbol  $\mathcal{R}$  denotes any of the quantities  $P$ ,  $R_{\mu\nu}$  or  $R$ .

Using the same procedure as before, we can obtain the CTP effective action coming from the dilaton loop, by replacing the Euclidean massive propagators by the CTP ones. These propagators can be evaluated using Riemann normal coordinates. To lowest order in the curvature, they coincide with the usual flat propagators. Therefore it is possible to use the properties between the Green functions without the curved spacetime problems mentioned before.

The CTP effective correction from the dilaton loop is given by

$$\begin{aligned} S_{eff}^{CTP}[\phi] = & -\frac{1}{2\pi} \int d^2x \int d^2y \Delta P(x) \Xi P(y) \int_0^1 d\gamma \operatorname{Re} G_{++}^\gamma(x, y) \theta(x^0 - y^0) \\ & + \frac{1}{2\pi} \int d^2x \int d^2y \Delta P(x) \Xi R(y) \int_0^1 d\gamma \frac{(1 - \gamma^2)}{2} \operatorname{Re} G_{++}^\gamma(x, y) \theta(x^0 - y^0) \\ & - \frac{1}{2\pi} \int d^2x \int d^2y \Delta R(x) \Xi R(y) \int_0^1 d\gamma \frac{(1 - 2\gamma^2 + 3\gamma^4)}{16} \operatorname{Re} G_{++}^\gamma(x, y) \theta(x^0 - y^0) \\ & + \frac{i}{4\pi} \int d^2x \int d^2y \Delta P(x) \Delta P(y) \int_0^1 d\gamma \operatorname{Im} G_{++}^\gamma(x, y) \\ & - \frac{i}{4\pi} \int d^2x \int d^2y \Delta P(x) \Delta R(y) \int_0^1 d\gamma \frac{(1 - \gamma^2)}{2} \operatorname{Im} G_{++}^\gamma(x, y) \\ & + \frac{i}{4\pi} \int d^2x \int d^2y \Delta R(x) \Delta R(y) \int_0^1 d\gamma \frac{(1 - 2\gamma^2 + 3\gamma^4)}{16} \operatorname{Im} G_{++}^\gamma(x, y), \end{aligned} \quad (62)$$

where with  $\Delta$  and  $\Xi$  we are denoting a half of the difference and sum, respectively, between the fields in each of the CTP branches; and where

$$G_{++}^\gamma(x, y) = \int \frac{d^2p}{(2\pi)^2} \frac{e^{ip(x-y)}}{p^2 + \frac{32\lambda^2}{1-\gamma^2} - i\epsilon}. \quad (63)$$

Owing to the global minus sign in the dilaton action, the CTP effective action for the dilaton has a global minus sign in its real part.

Finally, the total effective action including the dilaton loop can be written as Eq. (10) (that is an exact result) plus Eq. (62) (valid up to second order in powers of the curvature). Then, the absolute value of the decoherence functional has two contributions: the imaginary part of boundary terms in Eq. (13) plus the imaginary part of the dilaton effective action

$$|\mathcal{D}[\rho^+, \rho^-]| \approx e^{-\text{Im} [\text{boundary terms} + S_{eff}^\phi]}. \quad (64)$$

In particular, in the cosmological case and when the common hypersurface is defined by  $t = T$ , the boundary terms vanish, and Eq. (64) shows that decoherence still appears if one includes the dilaton loop in the evaluation of the effective action.

It is clear that a complete model should include the graviton loop too. Here we considered only the contribution of the dilaton. In the more realistic case where the quantum fluctuations coming from the metric are also included, an extra imaginary part will be obviously present, but our conclusion about the appearance of decoherence effects will be unaffected. It is also interesting to note that one can obtain the covariant semiclassical field equations taking the variation of action (62) with respect to the metric.

Conformal invariance can, of course, be broken in many other ways, for example by considering massive and/or non-minimally coupled matter fields. The classical action for such field is

$$S_f = -\frac{1}{2} \int d^2x \sqrt{g} [(\nabla f)^2 + (m^2 + \xi R)f^2], \quad (65)$$

which is similar to the action for the quantum fluctuations of the dilaton (Eq. (56)). The CTP effective action can be computed again using a covariant expansion in powers of the curvature, and one obtains a result similar to Eq. (62).

## VII. FINAL REMARKS

In this paper we have computed the influence functional for two dimensional models of dilaton gravity. When only conformal matter fields are quantized, the influence functional

can be computed exactly. This is, of course, a consequence of the conformal invariance. We have also shown that the influence functional depends strongly on the matching hypersurface. In particular, in the conformal gauge the influence action can be written as the difference between the Liouville actions for the metrics on the  $+/-$  branches plus an integral over  $\Sigma$ .

We used the influence action to derive the covariant form of the semiclassical field equations. These equations are real, causal and non-local, and become local in the conformal gauge. The derivation is non trivial due to the dependence of the propagators with the metric. It is not only of academic interest, since the procedure can be generalized to cases when conformal invariance is broken, i.e. when it is difficult to evaluate the  $\langle T_{\mu\nu} \rangle$  using conservation laws and the trace anomaly.

We have studied the quantum to classical transition in cosmological backgrounds. We have shown that the influence functional does not contain imaginary parts for some matching hypersurfaces. Therefore the semiclassical approximation is not valid in the conformal case.

We have also pointed out that the semiclassical approximation can be recovered by including the dilaton loop (and eventually quantum fluctuations of the metric), since in this case conformal invariance is broken. Two remarks are in order. The first one is that the quantum fluctuations of the dilaton also imply the validity of the semiclassical approximation for the dilaton background. This is important, since the “geometry” in the two dimensional models is determined by the metric and the dilaton. Moreover, when the two dimensional models are obtained by restricting the four dimensional Einstein-Hilbert action to metrics with spherical symmetry,  $\phi$  is part of the geometry since  $e^{-2\phi}$  is the radius of the 2-sphere. The second remark is that the dilaton and the metric loops are usually neglected by invoking the large  $N$  limit. However, we have seen that, no matter how large  $N$  is, there is no decoherence unless conformal invariance is broken. The dilaton and graviton loops are crucial in this sense.

It is worth to note that the existence of an imaginary part in the influence action is a necessary but not a sufficient condition for the validity of the semiclassical limit. In order to show semiclassicality we must have sufficient decoherence while maintaining classical

correlations. This requires a more detailed analysis that is beyond the scope of this paper.

In Section VI we have noted that the action for the dilaton fluctuations has a global minus sign with respect to the action for the matter fields. It would be interesting to investigate the effect of these fluctuations in the semiclassical field equations.

Finally, we would like to comment on related works about the validity of the semiclassical approximation in these models. In Refs. [23] and [26], this problem has been investigated in the vicinity of the black hole horizon. The main idea in those papers was the following. If the semiclassical approximation were a good one, the wave functional of the quantum fields should not depend very strongly on the black hole mass. Indeed, if we consider two different spacetimes, one describing the collapse of a black hole with mass  $M$ , the other with mass  $M + \Delta M$ , similar wave functionals at early times should not be “too different” after the black hole is formed (if  $\Delta M$  is sufficiently small). In order to compare both wave functionals, one can embed a spacelike hypersurface  $\Sigma$  in both spacetimes, and compute their inner product on  $\Sigma$ . It has been shown that, for certain hypersurfaces, this inner product is arbitrary small for the classical collapsing geometries [23], while it is of order one if quantum backreaction is included in the collapse [26].

The alert reader should have noticed that the inner product defined in Refs. [23] and [26] is exactly the influence functional  $e^{iS_{eff}^{CTP}}$ , evaluated at the geometries of collapsing black hole of masses  $M$  and  $M + \Delta M$ , with matching hypersurface  $\Sigma$ . The main technical difference between their calculation and ours is that they have worked in the Schrödinger picture, while we worked in the Heisenberg picture. In principle, all the results about the validity of the semiclassical approximation near the horizon of a black hole should be contained in the boundary terms that appear in the influence functional (Eq. (13)). Indeed, the result (13) is completely general, valid for any pair of metrics and any matching hypersurface. However, as we already pointed out, a complete analysis of this problem should include dilaton and metric fluctuations, since the semiclassical approximation in two dimensional models will be valid if both the metric and the dilaton decohere.

## ACKNOWLEDGMENTS

We would like to thank Bei Lok Hu and Miguel Ortiz for many useful discussions. This research was supported by Universidad de Buenos Aires, Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), and by Fundación Antorchas.

## REFERENCES

- [1] A. Strominger, Les Houches Lectures on Black Holes. Talk given at NATO Advanced Study Institute: Les Houches Summer School. Les Houches, France (1994). hep-th/9501071
- [2] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. **D45**, R1005 (1992)
- [3] J. G. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. **D46**, 344 (1992); Phys. Rev. **D47**, 533 (1993)
- [4] S. Bose, L. Parker, and Y. Peleg, Phys. Rev. **D54**, 7490 (1996)
- [5] J. P. Paz and S. Sinha, Phys. Rev. **D44**, 1038 (1991)
- [6] J. J. Halliwell, Phys. Rev. **D36**, 3626 (1987)
- [7] J. J. Halliwell, Phys. Rev. **D39**, 2912 (1989)
- [8] R. Lafamme and J. Louko, Phys. Rev. **D43**, 3317 (1991)
- [9] M. Gell-Mann and J. Hartle, in *Complexity, Entropy and Physics on Information*, edited by W. H. Zurek (Addison-Wesley, Reading, 1990); Phys. Rev. **D47**, 3345 (1993); T. Brun, Phys. Rev. **D47**, 3383 (1993); H. F. Dowker and J. J. Halliwell, Phys. Rev. **D46**, 1580 (1992)
- [10] R. Feynman and F. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963)
- [11] E. Calzetta and B. L. Hu, Phys. Rev. **D35**, 495 (1987); R. D. Jordan, Phys. Rev. **D33**, 444 (1986); E. Calzetta and B. L. Hu, Phys. Rev. **D40**, 656 (1989)
- [12] A. M. Polyakov, Phys. Lett. **B103**, 207 (1981)
- [13] E. Calzetta and B. L. Hu, Phys. Rev. **D49**, 6636 (1994)
- [14] F. C. Lombardo and F. D. Mazzitelli, Phys. Rev. **D53**, 2001 (1996)



- [15] E. Calzetta and F. D. Mazzitelli, Phys. Rev. **D42**, 4066 (1990)
- [16] F. Cooper, S. Habib, E. Mottola, J. P. Paz, and P. Anderson, Phys. Rev. **D50**, 2848 (1994)
- [17] F. C. Lombardo and F. D. Mazzitelli, Phys. Rev. **D55**, 3889 (1997)
- [18] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger in Ref. 2; S. Bose. L. Parker, and Y. Peleg in Ref. 4; Phys. Rev. Lett. **76**, 861 (1996)
- [19] S. M. Christensen and S. A. Fulling, Phys. Rev. **D15**, 2088 (1977)
- [20] B. S. DeWitt, in *Dynamical Theory of Groups and Fields*, Lectures at Les Houches Summer School of Theoretical Physics, published in the proceedings volume, *Relativity, Groups and Topology*, Gordon and Breach (1964)
- [21] N. D. Birrel and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge (1982)
- [22] F. D. Mazzitelli and J. G. Russo, Phys. Rev. **D47**, 4490 (1993)
- [23] E. Keski-Vakkuri, G. Lifschytz, S. D. Mathur, and M. E. Ortiz, Phys. Rev. **D51**, 1764 (1995)
- [24] A. Miković and V. Radovanović, Nucl. Phys. **B504**, 511 (1997); Class. Quant. Grav. **14**, 2647 (1997); hep-th/9706066
- [25] G. A. Vilkovisky, in *Quantum Theory of Gravity*, edited by S. M. Christensen (Hilger, Bristol, 1984); A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. **B282**, 163 (1987); **B333**, 471 (1990); I. G. Avramidi, Yad. Fiz. **49**, 1185 (1989) [Sov. J. Nucl. Phys. **49**, 735 (1989)]
- [26] S. Bose. L. Parker, and Y. Peleg, Phys. Rev. **D53**, 7089 (1996)